# Examiners' Report Principal Examiner Feedback 

## Summer 2017

Pearson Edexcel International A-Level in Further Pure Mathematics (WFM01/01)

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Publications Code WFM01_01_1706_ER
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## IAL Mathematics Unit Further Pure 1 <br> Specification WFM01/ 01

## General Introduction

This paper was accessible and there was plenty of opportunity for a typical E grade student to gain some marks across all of the questions. There were some testing questions involving mathematical induction, the application of finite series summations, complex numbers and matrices that allowed the paper to discriminate well between the higher ability levels.

In summary, Q1, Q2, Q4, Q5, Q6(a), Q7(a), Q7(b), Q8(a) and Q9(a) were a good source of marks for the average student, mainly testing standard ideas and techniques and Q3, Q6(b), Q8(b), Q8(c), Q8(d), Q9(b), Q9(d), Q10(d) and Q10(e) were discriminating at the higher grades. Q10(e) proved to be the most challenging question on the paper.

## Report on Individual Questions

## Question 1

This question proved accessible with the majority of students scoring full marks.
The most common mistakes made by students in this question were not recalling that the sum and product of roots in a quadratic equation $a x^{2}+b x+c=0$ were $-\frac{b}{a}$ and $\frac{c}{a}$ respectively; not correctly deducing the result $\alpha^{2}+\beta^{2} \equiv(\alpha+\beta)^{2}-2 \alpha \beta$ and manipulating $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ incorrectly to give $\frac{\alpha+\beta}{\alpha \beta}$. Many students who correctly stated $\alpha+\beta=\frac{5}{3}$ and $\alpha \beta=\frac{1}{3}$, generally applied the correct method and went on to find the correct value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$. A small minority of students, who did not obey the instructions given in the question, applied $\alpha, \beta=\frac{5 \pm \sqrt{13}}{6}$ and so gained no marks for this question.

## Question 2

Part (a) was accessible to almost all students, and there were many fully correct answers. Almost all students obtained a $2 \times 2$ matrix. For a minority of students, errors occurred in simplifying the elements, which subsequently led to the loss of at least one mark in part (b).

In part (b), the majority of students applied a correct method to find the determinant, with many finding a correct value for $k$. Few students made sign errors when manipulating their equation to find the final answer despite all previous work being correct.

## Question 3

This question was generally answered well. A significant number of students failed to show that for $n=1$, both the LHS and RHS of the general statement were both equal to $\frac{1}{3}$. Some students did not produce any creditable work after showing that the basis case was true. Most students correctly added the $(k+1)^{\text {th }}$ term to the sum of $k$ terms to give $\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)}=\frac{1}{2}-\frac{1}{(k+1)(k+2)}+\frac{2}{(k+1)(k+2)(k+3)}$.

Some students manipulated this to give $\sum_{r=1}^{k+1} \frac{2}{r(r+1)(r+2)}=\frac{1}{2}-\frac{1}{(k+2)(k+3)}$, with a significant number fudging this correct result from incorrect intermediate work.

Some students did not bring all strands of their proof together to give a fully correct proof. A minimal acceptable proof, following on from completely correct work, would incorporate the following parts: assuming the general result is true for $n=k$; then showing the general result is true for $n=k+\mathbf{1}$; showing the general result is true for $n=1$; and finally concluding that the general result is true for all positive integers.

## Question 4

This question proved accessible with the majority of students scoring full marks.
In part (a), most students substituted $x=4 t, y=\frac{4}{t}$ into $3 y-2 x=10$ and manipulated their equation to form a quadratic equation in $t$. Some students, who expressed $H$ in Cartesian form, substituted $3 y-2 x=10$ into $x y=16$ and manipulated their result to form a quadratic equation in either $x$ or $y$. Most students used a correct method for solving their quadratic equation and progressed to find the coordinates for both $A$ and $B$. Common errors included using an incorrect method for solving their quadratic equation and not assigning their coordinates to $A$ and $B$ correctly.

In part (b), many students used their $A\left(x_{1}, y_{1}\right)$ and their $B\left(x_{2}, y_{2}\right)$ in a correct method of applying $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$ to find the midpoint of $A B$. A significant number, however, applied the incorrect formula $\left(\frac{x_{1}-x_{2}}{2}, \frac{y_{1}-y_{2}}{2}\right)$.

## Question 5

This question was generally accessible to those students who were familiar with the numerical methods that were being tested.

In part (a), it was clear in many students' well presented solutions that they were applying a fully correct method for interval bisection with a significant number presenting their method in a table. Some students should be aware that values of the function are required in their solution. In this case, for example, values for at least one of either $f(2)$ or $f(2.1)$ and values for both $f(2.05)$ and $f(2.025)$ are required to justify their final interval. A noticeable number, however, did not produce a correctly stated interval of $(2.025,2.050)$, with some giving no statement at all or just a single $x$ value. Students should be made aware that, if the interval notation is used, the smaller number should be written first.

In part (b), many students differentiated $\mathrm{f}(x)$ correctly and applied the NewtonRaphson procedure correctly to give a second approximation for $\alpha$ as 2.04. A few
students differentiated $-\frac{7}{\sqrt{x}}$ incorrectly to give either $-7 x^{-2},-14 x^{\frac{1}{2}}$ or $-\frac{14}{3} x^{\frac{3}{2}}$. In some cases, a lack of working did mean that it was sometimes difficult for examiners to determine whether the procedure was applied correctly.

## Question 6

This question was accessible with the majority of students scoring full marks.

In part (a), almost all students expanded the cubic expression $r^{2}(r+1)$ and substituted the standard formulae for $\sum_{r=1}^{n} r^{3}$ and $\sum_{r=1}^{n} r^{2}$ into $\sum_{r=1}^{n}\left(r^{3}+r^{2}\right)$. Students who directly factorised out $\frac{1}{12} n(n+1)$ were generally more successful in obtaining the correct answer. A number of students expanded to give a correct $\frac{1}{4} n^{4}+\frac{5}{6} n^{3}+\frac{3}{4} n^{2}+\frac{1}{6} n$, and then generally struggled to obtain the correct fully factorised answer.

Part (b) was found to be more demanding when compared to part (a). It was pleasing, however, that the majority of students applied $\sum_{r=25}^{49}\left(r^{2}(r+1)+2\right)$ to give $\mathrm{f}(49)-\mathbf{f}(24)+98-48$, where $\mathrm{f}(n)=\frac{1}{12} n(n+1)(n+2)(3 n+1)$ and obtained the correct answer of 1446200 . Unsuccessful attempts included applying either $\mathbf{f}(49)-\mathbf{f}(24)+2, \mathbf{f}(49)-\mathbf{f}(25)+98-50, \quad \mathbf{f}(49)-\mathbf{f}(24)+2, \mathbf{f}(49)-\mathbf{f}(24)$ or even $\left(49^{2}(50)+2\right)-\left(24^{2}(25)+2\right)$.

## Question 7

This question proved accessible with the majority of students scoring full marks.
In part (a), almost all students wrote down the complex conjugate root $1-2 \mathrm{i}$.
In part (b)(i), most students used the conjugate pair to write down and multiply out $(z-(1+2 \mathrm{i}))(z-(1-2 \mathrm{i}))$ in order to identify the quadratic factor $z^{2}-2 z+5$. Some students achieved this quadratic factor using the sum and product of roots method. Most students used algebraic long division to establish the other quadratic factor, although some students used a method of comparing coefficients. Those students who had progressed this far either applied the quadratic formula or completed the square to find the 2 other roots of their $z^{2}+6 z+13=0$. A number of sign errors or manipulation errors were seen in this part.

In part (b)(ii), many students stated $a$ correctly as 65 . Common incorrect values for $a$ included $-65,5$ or 1 .

## Question 8

This was a well-answered question with a significant number of students scoring full marks.
In part (a), students used a variety of methods to find $\frac{\mathrm{d} y}{\mathrm{~d} x}$. Most students wrote $y$ in terms of $x$ and proceeded to find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ correctly in terms of $p$. Some students used implicit differentiation or the chain rule with parametric equations. Most students were then successful in finding the equation of the tangent and obtained the given result in part (a). Few students did not use a calculus method to find $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{1}{p}$, and so lost marks in part (a).

In part (b), many students stated $a$ correctly as 9 . Common incorrect values for $a$ included $-9,-1$ or 1 .

In part (c), many students substituted $x=-a$ from part (b) and $y=6$ into the given tangent equation and manipulated their result to form a quadratic equation in $p$. Most students used the quadratic formula to solve their equation and rejected the negative root for $p$, with a large proportion stating the value of the simplified surd $\frac{1+\sqrt{10}}{3}$. Common errors included substituting $y^{2}=36 x$ into the tangent equation, substituting $(a, 6)$ or $(a, 0)$ into the tangent equation or stating two values for $p$ as their final answer.

In part (d), most students substituted their value of $p$ into $P\left(9 p^{2}, 18 p\right)$ and generally used their calculator to find each coordinate as a simplified surd. Some students, not realising $p>0$, gave two sets of coordinates for $P$.

## Question 9

This question discriminated well between students of all abilities.
In part (a), most students applied Pythagoras correctly to find a correct exact value for the modulus of $z$. A small minority subtracted rather than added the two squared terms. A significant number of students did not realise that the required angle was in the fourth quadrant and many of them found an angle in the first quadrant. Most students worked in radians but there was a significant minority who gave their answers in degrees.

In part (b), many students rearranged the equation $w z=\lambda i$ to make $w$ the subject and
substituted for $z$ to give $w=\frac{\lambda i}{\left(\frac{1}{5}-\frac{2}{5} \mathrm{i}\right)}$. A large proportion, proceeded to multiply the numerator and denominator of the right hand side of the equation by $\frac{1}{5}+\frac{2}{5} \mathrm{i}$ and a significant number proceeded to find a correct $w=-2 \lambda+\lambda i$. A few students started this question by rewriting $w z=\lambda \mathrm{i}$ as $\left(\frac{1}{5}-\frac{2}{5} \mathrm{i}\right)(a+b \mathrm{i})=\lambda \mathrm{i}$. These students equated the real part to write an equation in terms of $a$ and $b$; and equated the imaginary part to write a second equation in terms of $a, b$ and $\lambda$. Only a small proportion solved these simultaneous equations to find a correct $w$.

In part (c), most students used a correct method of substituting $z, \lambda=\frac{1}{10}$ and their $w$ into the expression $\frac{4}{3}(z+w)$. Only the more able students correctly evaluated this expression.

In part (d), while most students plotted $z$ correctly on their Argand diagram, only the more able students correctly plotted and labelled the other points representing $z w, w$ and $\frac{4}{3}(z+w)$. A significant number of students drew ridiculously small diagrams and in a number of cases it was difficult for examiners to decipher the information contained in these diagrams.

## Question 10

This question discriminated well between students of all abilities. A significant minority, however, made no creditable progress in this question.

The majority of students found correct matrices for $\mathbf{P}$ and for $\mathbf{Q}$, although a few expressed their final answers for these matrices in terms of sine and cosine. Most students attempted to multiply their matrices in part (c), with some making sign errors in their final simplified matrix for $\mathbf{R}$.

A significant number of students struggled to gain full marks in part (d). Although most stated that a rotation was involved, some did not state the centre of rotation. Errors included incorrectly stating the sense of the rotation or stating an incorrect angle of rotation.

Many students found part (e) challenging, and seemed unfamiliar with the style of question. Some students clearly used their calculator to state $\sin 75^{\circ}=\frac{\sqrt{2}+\sqrt{6}}{4}$, followed by $\sin 75^{\circ}=\frac{\sqrt{3}+1}{2 \sqrt{2}}$, and made no justification for the link between $\sin 75^{\circ}$ and $\sin 105^{\circ}$. Also, a significant number of students used their calculator to state the exact value for $\cos 75^{\circ}$ as $\frac{\sqrt{6}-\sqrt{2}}{4}$ with no justification for this value.

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